Week 33: 4/18-4/22 Math I Due: 4/25

Objectives:

1. To review contents of Chapter 5.
2. To assess knowledge of Chapter 5.
3.
4.

Monday: **In Class:** **Review Sections 5-1, 5-2, and 5-3.** **Homework:** **Complete 5-1, 5-2, and 5-3 Review Handouts**
Tuesday: **In Class:** **Review Sections 5-4, 5-5, and 5-8** **Homework:** **Complete 5-4, 5-5, and 5-8 Review Handouts**
Wednesday: **Homework:** **Study for Chapter 5 Test**
Thursday: **In Class:** **Chapter 5 Test** **Homework:** **None**
Friday: **Homework:** **Complete "Getting Ready for Chapter 12" in Volume 2.** **BRING VOLUME 2 BOOKS TO CLASS NEXT WEEK!!!!!**

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Reteaching 5-1

Zero and Negative Exponents

For every nonzero number a, $a^0 = 1$.

For every nonzero number a and integer n, $a^{-n} = \frac{1}{a^n}$. In other words, when the exponent is negative, raise the reciprocal of the base to the opposite of the exponent.

Problem

What is the simplified form of each expression?

a. $3.9^0 = 1$ — Since the exponent is 0 but the base of the expression is 3.9, which is not 0, the expression has a value of 1.

b. $9^{-2} = \frac{1}{9^2}$ — The exponent is negative, so raise the reciprocal of 9, or $\frac{1}{9}$, to the exponent $-(-2)$, or 2.

$= \frac{1}{81}$ — Simplify.

Problem

What is the simplified form of $\frac{7b^{-3}}{a^2}$ using only positive exponents?

$\frac{7b^{-3}}{a^2} = \frac{7}{a^2} \cdot b^{-3}$ — Rewrite the expression as a product of factors with positive exponents and factors with negative exponents.

$= \frac{7}{a^2} \cdot \frac{1}{b^3}$ — Rewrite the factor with the negative exponent by raising the reciprocal of the base to a positive exponent.

$= \frac{7}{a^2 b^3}$ — Simplify by multiplying.

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Reteaching (continued) 5-1

Zero and Negative Exponents

Exercises

Write each expression as an integer, a simple fraction, or an expression that contains only positive exponents. Simplify.

1. 2.3^0
2. 10^{-4}
3. $2a^{-5}$
4. 113.7^0
5. 19^{-1}
6. $\frac{3^{-3}}{p}$
7. $(7q)^{-1}$
8. $\left(-\frac{7}{8}\right)^{-2}$
9. $1.8c^0$
10. $(-9.7)^0$

Write each expression so that it contains only positive exponents. Simplify.

11. -6^{-3}
12. $-2rs^{-5}$
13. $7x^{-8}y^0$
14. $\left(\frac{5a}{3b}\right)^{-2}$
15. $(-8v)^{-2}w^3$
16. $\frac{2^{-3}}{m^0n^{-1}}$
17. $(3xy)^0z$
18. $\frac{-3^{-3}}{uv^{-2}}$

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Reteaching 5-2

Exponential Functions

Functions that can be modeled by an equation of the form $y = a \cdot b^x$ are exponential functions. These functions have properties that are different from the properties of linear and quadratic functions.

Consider the three function tables below.

x	y
0	3
1	9
2	27
3	81
4	243
5	729

x	y
0	3
1	6
2	12
3	24
4	48
5	96

x	y
0	2
1	10
2	50
3	250
4	1250
5	6250

Notice that in each table, the x-value increases by a constant amount. If these functions were linear, the y-values would also increase by a constant amount. You used these values to find the slope of linear equations.

This does not hold true for exponential functions. See if you can determine the property that holds true for all exponential functions by:

a. Finding the sum of some random pairs of consecutive y-values in each table.

b. Finding the difference between some random pairs of consecutive y-values in each table.

c. Finding the product of some random pairs of consecutive y-values in each table.

d. Finding the quotient of some random pairs of consecutive y-values in each table.

You should have noticed that, for each table, the quotients remain the same.

Exponential functions model an initial amount, a, that is repeatedly multiplied by the same positive number, b. The number of times the multiplication occurs is determined by the independent variable, x, which is the exponent in the power b^x.

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Reteaching (continued) 5-2

Exponential Functions

1. For each of the tables on the previous page, extend them two units in each direction. Use the common difference in the x-values and the common ratio in the y-values to do the extension. The first table is done for you.

x	y
−2	$\frac{1}{3}$
−1	1
0	3
1	9
2	27
3	81
4	243
5	729
6	2187
7	6561

x	y
0	3
1	6
2	12
3	24
4	48
5	96

x	y
0	2
1	10
2	50
3	250
4	1250
5	6250

2. Plot the points in each of your extended tables on separate coordinate grids. Connect the points with a smooth curve. The domain of each function is all real numbers and that the range is all positive real numbers. Explain why there are negative values for x but not for y.

3. For each of the tables, identify the starting value a and the common ratio b. For the first table, a is 1 and b is 3. Next, write the exponential function that describes each table. The function for the first table is $f(x) = 1 \cdot 3^x$. Check if your function is correct by substituting in x-values and seeing if the function produces values for y that match the values in the table.

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Reteaching 5-3

Comparing Linear and Exponential Functions

Suppose all the x-values in a table have a common difference. If all the y-values have a common difference, then the table represents a linear function. If all the y-values have a common ratio, then the table represents an exponential function.

Problem

Does the table represent a linear function or an exponential function? Explain.

a.

	x	y	
+1	0	3	× 3
+1	1	9	× 3
+1	2	27	× 3
+1	3	81	× 3
+1	4	243	× 3
	5	729	

The difference between each x-value is 1 and the ratio between each y-value is 3. The table represents an exponential function because there is a common difference between x-values and a common ratio between y-values.

b.

	x	y	
+1	0	3	+ 3
+1	1	6	+ 3
+1	2	9	+ 3
+1	3	12	+ 3
+1	4	15	+ 3
	5	18	

The difference between each x-value is 1 and the difference between each y-value is 3. The table represents a linear function because there is a common difference between x-values and a common difference between y-values.

The average rate of change of a function $f(x)$ over the interval $a \le x \le b$ is given by the following formula.

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

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Reteaching (continued) 5-3

Comparing Linear and Exponential Functions

Problem

What is the average rate of change for the function $f(x) = 2 \cdot 2^x$ over the intervals $-2 \leq x \leq 0$, $0 \leq x \leq 2$, and $2 \leq x \leq 4$? Describe what you observe.

Step 1 Make a table of values.

x	-2	-1	0	1	2	3	4
$f(x)$	0.5	1	2	4	8	16	32

Step 2 Find the average rates of change over the intervals $-2 \leq x \leq 0$, $0 \leq x \leq 2$, and $2 \leq x \leq 4$.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{2 - 0.5}{2} = 0.75$$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(0)}{2 - 0} = \frac{8 - 2}{2} = 3$$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(4) - f(2)}{4 - 2} = \frac{32 - 8}{2} = 12$$

The average rate of change is different over each interval. It increases with each interval. So the rate of change is increasing.

Exercises

1. Does the table represent a *linear function* or an *exponential function*? Explain.

x	0	1	2	3
y	15	20	25	30

2. Suppose that the fish population of a lake decreases by half each week. Can you model the situation with a linear function or an exponential function? Explain.

3. Find the average rate of change for the function $y = 2.5 \cdot 2^x$ over the intervals $1 \leq x \leq 3$, $3 \leq x \leq 5$, and $5 \leq x \leq 7$. Describe what you observe.

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Reteaching 5-4

Exponential Growth and Decay

Exponential functions can model the growth or decay of an initial amount.

The basic exponential function is $y = a \cdot b^x$ where

a represents the initial amount

b represents the growth (or decay) factor. The growth factor equals 100% plus the percent rate of change. The decay factor equals 100% minus the percent rate of decay.

x represents the number of times the growth or decay factor is applied.

y represents the result of applying the growth or decay factor x times.

Problem

A gym currently has 2000 members. It expects to grow 12% per year. How many members will it have in 6 years?

There are 2000 members to start, so $a = 2000$.
The growth per year is 12%, or 0.12, so $b = 1 + 0.12$ or 1.12.
The desired time period is 6 years, so $x = 6$.
The function is $y = 2000 \cdot 1.12^x$.

When $x = 6$, $y = 2000 \cdot 1.12^6 \approx 3948$. So, the gym will have about 3948 members in 6 years.

In Exercises 1–3, identify *a*, *b*, and *x*. Then use them to write the exponential function that models each situation. Finally use the function to answer the question.

1. When a new baby is born to the Johnsons, the family decides to invest $5000 in an account that earns 7% interest as a way to start the baby's college fund. If they do not touch that investment for 18 years, how much will there be in the college fund?

2. The local animal rescue league is trying to reduce the number of stray dogs in the county. They estimate that there are currently 400 stray dogs and that through their efforts they can place about 8% of the animals each month. How many stray dogs will remain in the county 12 months after the animal control effort has started?

3. A basket of groceries costs $96.50. Assuming an inflation rate of 1.8% per year, how much will that same basket of groceries cost in 20 years?

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Reteaching (continued) 5-4

Exponential Growth and Decay

While it is usually fairly straightforward to determine *a,* the initial value, you often need to read the problem carefully to make sure that you are correctly identifying *b* and *x.* This is especially true when considering situations where the given growth rate is applied in intervals that are not the same as the given value of *x.*

Problem

You invest \$1000 in an account that pays 8% interest. If nothing changes, $a = 1000$, $b = 1.08$, and $x = 3$. The function is $y = 1000 \cdot 1.08^3$. How much money will you have left after 3 years?

In this case, the interest is *compounded annually,* meaning it is added to your account at the end of each year. But what happens if the interest is *compounded quarterly,* meaning it is added to your account at the end of each quarter of a year?

There will be 12 compounding periods in the 3-year period.

In each compounding period you add the appropriate fraction of the total annual interest, in this case one-fourth of the interest.

The new function is $y = 1000 \cdot \left(1 + \frac{0.08}{4}\right)^{12}$, given that *a* is the initial investment, *b* is the amount of growth for each compounding period, and *x* is the number of compounding periods.

Compounded annually, the value of the investment will be \$1259.71 after 3 years. Compounded quarterly, the value will be \$1268.24.

In Exercises 4 and 5, identify *a, b,* and *x.* Then write the exponential function that models the situation. Finally, use the function to answer the question.

4. You invest \$2000 in an investment that earns 6% interest, compounded quarterly. How much will the investment be worth after 5 years?

5. You invest \$3000 in an investment that earns 5% interest, compounded monthly. How much will the investment be worth after 8 years?

6. The formula that financial managers and accountants use to determine the value of investments that are subject to compounding interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where *A* is the final balance, *P* is the initial deposit, *r* is the annual interest rate, *n* is the number of times the interest is compounded per year and *t* is the number of years. Redo Exercises 4 and 5 using this formula.

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Reteaching 5-5

Solving Exponential Equations

Recall that *exponential equations* are equations that have variables in an exponent:

exponent

$$y = a \cdot b^x$$

base

In this lesson, two methods were shown for solving exponential equations:

Algebraic First rewrite the equation so that each side has the same base. Then use the fact that if the base $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Graphical Graph each side of the equation. The solution is the x-coordinate of the point of intersection of the two graphs.

Problem

What is the solution of the exponential equation $3^{x-2} = 27$?

Method 1 Solve algebraically. Rewrite the equation so that each side has the same base. Then set the exponents equal to each other and solve for the variable.

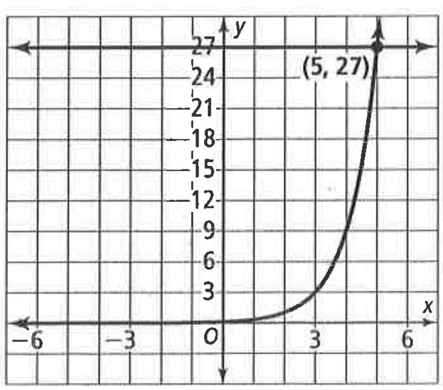
$3^{x-2} = 27$

$3^{x-2} = 3^3$ Rewrite 27 as a power with base 3.

$x - 2 = 3$ The bases are the same, so the exponents are equal.

$x = 5$ Add 2 to both sides to solve for x.

Method 2 Solve graphically. Graph each side of the equation. Identify the point of intersection.



The graphs intersect at the point (5, 27), so the solution is $x = 5$.

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Reteaching (continued) 5-5

Solving Exponential Equations

Exercises

Solve each exponential equation.

1. $5^x = 125$

2. $5^x = \frac{1}{125}$

3. $\frac{1}{36} = 6^{x+5}$

4. $2^{3x} = \frac{1}{64}$

5. Find the solution of the equation $8 = 2^{3x-1}$ using both methods.
   a. Solve algebraically. Explain each step.

   b. Solve graphically. Justify your solution.

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Reteaching 5-8

Simplifying Radicals

You can remove perfect-square factors from a radicand.

Problem

What is the simplified form of $\sqrt{80n^5}$?

In the radicand, factor the coefficient and the variable separately into perfect square factors, and then simplify. Factor 80 and n^5 completely and then find paired factors.

Solve

$80 = 8 \cdot 10 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$	Factor 80 completely.
$= (2 \cdot 2)(2 \cdot 2) \cdot 5 = (2 \cdot 2)^2 \cdot 5$	Find pairs of factors.
$\sqrt{80} = \sqrt{4^2 \cdot 5} = \sqrt{4^2} \cdot \sqrt{5}$	Use the rule $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
$= 4 \cdot \sqrt{5} = 4\sqrt{5}$	The square root of a number squared is the number: $\sqrt{a^2} = a$.
$n^5 = n \cdot n \cdot n \cdot n \cdot n$	Factor n^5 completely.
$= (n \cdot n) \cdot (n \cdot n) \cdot n = (n \cdot n)^2 \cdot n$	Find pairs of factors.
$\sqrt{n^5} = \sqrt{(n \cdot n)^2} \cdot \sqrt{n}$	Separate the factors.
$= n^2 \cdot \sqrt{n} = n^2\sqrt{n}$	Remove the perfect square.
$\sqrt{80n^5} = 4 \cdot n^2\sqrt{5 \cdot n} = 4n^2\sqrt{5n}$	Combine your answers.

Check

$\sqrt{80n^5} \stackrel{?}{=} 4n^2\sqrt{5n}$	Check your solution.
$\dfrac{\sqrt{80n^5}}{\sqrt{5n}} \stackrel{?}{=} \dfrac{4n^2\sqrt{5n}}{\sqrt{5n}}$	Divide both sides by $\sqrt{5n}$.
$\sqrt{16n^4} \stackrel{?}{=} 4n^2$	Simplify.
$4n^2 = 4n^2$ ✓	

Solution: The simplified form of $\sqrt{80n^5}$ is $4n^2\sqrt{5n}$.

Exercises

Simplify each radical expression.

1. $\sqrt{100n^3}$
2. $\sqrt{120b^4}$
3. $\sqrt{66t^5}$
4. $\sqrt{32x}$
5. $\sqrt{525c^7}$
6. $\sqrt{86t^2}$
7. $\sqrt{50g^3}$
8. $\sqrt{54h^6}$
9. $\sqrt{35y}$

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Reteaching (continued) 5-8

Simplifying Radicals

Problem

What is the simplified form of $\sqrt{\frac{27t^4}{48t^2}}$?

Begin by cancelling the common factors in the numerator and denominator. Simplify the numerator and denominator separately when the denominator is a perfect square. Remember that the radical is not simplified if there is a radical in the denominator. Multiply to remove the radical from the denominator.

Solve

$$\sqrt{\frac{27t^3}{48t^4}} = \sqrt{\frac{3 \cdot 3 \cdot 3 \cdot t \cdot t \cdot t}{3 \cdot 4 \cdot 4 \cdot t \cdot t \cdot t \cdot t}}$$

Factor the numerator and denominator completely.

$$= \sqrt{\frac{\cancel{3} \cdot 3 \cdot 3 \cdot \cancel{t} \cdot \cancel{t} \cdot \cancel{t}}{\cancel{3} \cdot 4 \cdot 4 \cdot \cancel{t} \cdot \cancel{t} \cdot \cancel{t} \cdot t}}$$

Cancel the common factors.

$$= \frac{\sqrt{(3 \cdot 3)}}{\sqrt{(4 \cdot 4)t}} = \frac{\sqrt{3^2}}{\sqrt{4^2 t}}$$

Find pairs of factors. These are the perfect-square factors.

$$= \frac{3}{4\sqrt{t}}$$

Simplify the numerator and denominator separately to remove the perfect-square factors. $\sqrt{3^2} = 3$ and $\sqrt{4^2 t} = 4\sqrt{t}$

$$= \frac{3}{4\sqrt{t}} \frac{(\sqrt{t})}{(\sqrt{t})}$$

Multiply the numerator and denominator by $\sqrt{t}$ to remove $\sqrt{t}$ from the denominator.

$$= \frac{3\sqrt{t}}{4\sqrt{t \cdot t}} = \frac{3\sqrt{t}}{4\sqrt{t^2}} = \frac{3\sqrt{t}}{4t}$$

Remove the perfect-square factor from the denominator.

Solution: The simplified form of $\sqrt{\frac{27t^3}{48t^4}}$ is $\frac{3\sqrt{t}}{4t}$.

Exercises

Simplify each radical expression.

10. $\sqrt{\frac{49}{81}}$

11. $\sqrt{\frac{18x^4}{200}}$

12. $\sqrt{\frac{28s}{s^3}}$

13. $\sqrt{\frac{25a^5}{9a^7}}$

14. $\sqrt{\frac{40b^4}{12b^3}}$

15. $\sqrt{\frac{48}{6t^6}}$

16. $\sqrt{\frac{50z^3}{4x^2}}$

17. $\sqrt{\frac{t^5}{64}}$

18. $\sqrt{\frac{32t}{t}}$